Energy approach of thresholds stresses in multi-axial fatigue under dissymmetrical loading

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Abstract The criterion established by energy approach and suggested in literature is based on the concept influence volume around the "critical point" requested by the threshold stress and uses like parameter the voluminal density of the energy deformation provided by cycle of each element of volume. In this context, we propose an analytical study improved on cylindrical test-tubes of fatigue, based on the analysis of the total energy variation. By preserving the concept of threshold stress of "not damage" lower than the limit of conventional endurance of material, this proposal highlights initiating stress and propagating stress threshold of cracks. It thus envisages the effects of the loadings types on the resistance to fatigue of the parts under combined requests (torsion and cross-bending, torsion and rotational bending) and highlighting the influence of the average stress.

1 INTRODUCTION

This paper is interested in the loadings with non null average value and uses like parameter of damage the average, on a cycle of loading, voluminal density of elastic deformation energy brought into play after material adaptation. In structure mechanics, we are devoted to various questions concerning the energy behavior of the test-tubes by marrying the energy and analytical methods. On the basis of the quantity of elastic strain work, it is proceeded to an approach in threshold of the characterization of the optimal energy variation. We will have thus to carry out an energy study on test-tubes cylindrical of fatigue subjected to the dissymmetrical loads. Curves of analysis of the results are designed to validate the new concept suggested. It is shown then that the use of the energy variation makes it possible to determine with a good approximation two thresholds stresses responsible for initiating crack and crossing crack in the case of the rotational bending (respec. cross-bending) combined with the torsion applied to the test-tubes of cylindrical section.

2 ENERGY OF ELASTIC STRAIN IN A STATE OF UNIAXIAL LOADING

Around points potentially critical Ci (where a fatigue crack can start), Palin-Luc and Lasserre [8] consider that there is a surface of S*(Ci) influence and introduce the quantity $\varpi_f(C_i)$ like the surface average of the elastic strain energy quantity provided, able to initiate microscopic cracks:

$$\varpi_f(C_i) = \frac{1}{S^*(C_i)} \int_{S^*(C_i)} \left[W_f(M) - W_f^*(C_i) \right] ds \tag{1.1}$$

Several concepts were proposed in the literature to improve the forecasts of the stress: the gradient stress [1] or the layer criticizes [2] for example. However, according to authors none of these concepts explains the differences of the limits of endurance observed in traction, rotational bending and cross-bending. This is why, according to authors, it is important to reason on a cycle of loading like that was proposed by Tsybanev [10], Froustey [6], Banvillet and Pallin-luc [4]. So that, the provided energy of elastic strain W_f defined in any point M of the part by:

$$W_f(M) = \int_0^T \sigma_{ij}(M,t)\, \dot{\varepsilon}^e_{ij}(M,t)\; dt \tag{1.2}$$

For uniaxial states of stresses, in extreme cases of endurance, the author postulates that the quantity $\varpi_f(C_i)$ is an intrinsic size with material noted $\varpi^D_{f\ uniax}$. It thus does not depend on the request (traction, rotational bending and cross-bending), of or the equation: $\varpi^D_{f\ Uniax} = \varpi^D_{f\ Trac} = \varpi^D_{f\ Frot} = \varpi^D_{f\ Fpla}$

(1.3)

3 DEFORMATION ENERGY ACCORDING TO THE THRESHOLD STRESS OF COMBINED LOADINGS.

We consider in the continuation the following relations (respectively) of the equivalent stresses of rotational bending $\sigma_{Eq,Frot}$, traction $\sigma_{Eq,Trac}$ and the dissymmetrical loads $\sigma^{*}_{Eq,Dissy}$

$$\sigma^2_{Eq,Frot} = \sigma^2_{m,Frot} + \sigma^2_{Frot}, \ \sigma^2_{eq,Trac} = \sigma^2_{m,Trac} + \sigma^2_{Trac} \text{ and } \sigma^{*}_{eq,Dissy} = \sigma^{*}_{m,Trac} + \sigma^{*}_{Dissy} \quad (1.4)$$

The procedure of calculation of the deformation energy is thus summarized from now on in the following way. We distinguish two levels: the level with an elementary test of rupture, where one reasons on a symmetrical load, and the level of the combined loads, where we propose an analytical study of the variation of deformation energy expressed according to the thresholds stress of the part.

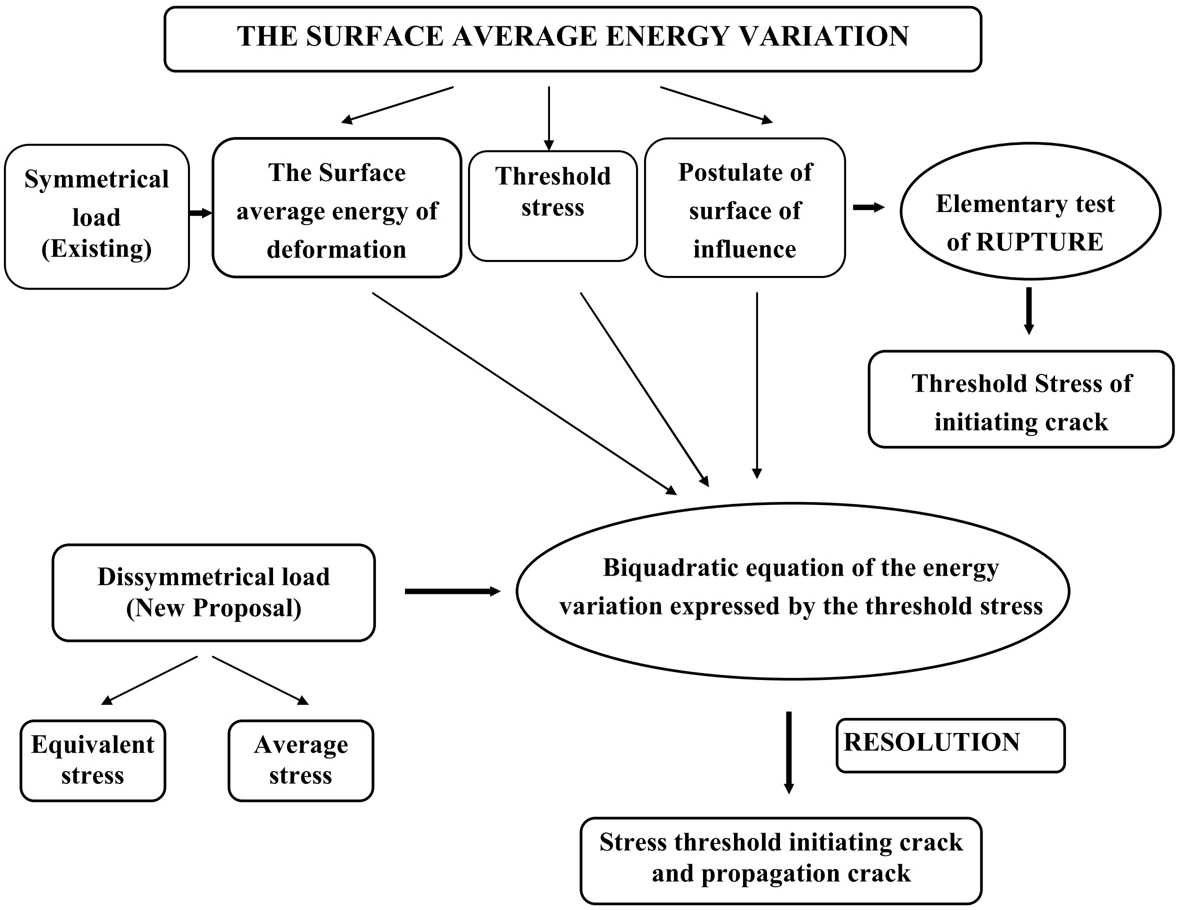


Fig 1.1: Synoptic general of the fatigue behaviour

3.1 Dissymmetrical rotational bending

The data necessary are $\varpi_{fTrac} = \frac{1}{E}\left(\sigma^2_{Trac} - \sigma^{*\,2}_{Dis}\right)$ and in the case of the loading checking the condition $\frac{a^*}{R} < 1$, Banvillet and al. [4] shows that:

$$\varpi_{fFrot} = \frac{1}{4E\left[1-\left(\frac{b^*}{R}\right)\left(\frac{a^*}{R}\right)\right]}\left\{(\sigma^2_{m,Frot}+3\sigma^2_{Frot})-\left\{\sigma^2_{Frot}\left(\frac{b^*}{R}\right)\left(\frac{a^*}{R}\right)^3+\sigma^2_{Eq,Frot}\left(\frac{b^*}{R}\right)^3\left(\frac{a^*}{R}\right)\right\}\right\}-\frac{2\sigma^2_{Trac,-1}-\sigma^2_{Frot,-1}}{E}$$

(1.5)

We have: $$\mathrm{W}^*_{\mathrm{fFrot}} = \frac{\sigma^2_{m,Frot}}{E}\cdot\left(\frac{y^*}{R}\right)^2+\frac{\sigma^2_{Frot}}{E}\cdot\left(\frac{r^*}{R}\right)^2 \quad (1.6),$$

$$\mathrm{W}^*_{\mathrm{fTrac}} = \frac{1}{E}\left(\sigma^2_{m,Trac}+\sigma^{*\,2}_{Dissy}\right) \text{ where } y^* = r^*\cos\theta \quad (1.7)$$

With the constraint threshold the postulate of the volume of influence $\mathrm{W}^*_{\mathrm{Trac}} = \mathrm{W}^*_{\mathrm{Frot}}$ makes it possible to write : $$\frac{\sigma^2_{m,Trac}+\sigma^{*\,2}_{Dissy}}{\sigma^2_{m,Frot}\cos^2\theta+\sigma^2_{Frot}} = \left(\frac{r^*}{R}\right)^2 \quad (1.8)$$

The Iso-énergetic level (1.8) is described by an ellipse on the cross-section of the test-tube, illustrated by Fig 1.2. The small half axis a^* and the half large axis b^* of this ellipse.

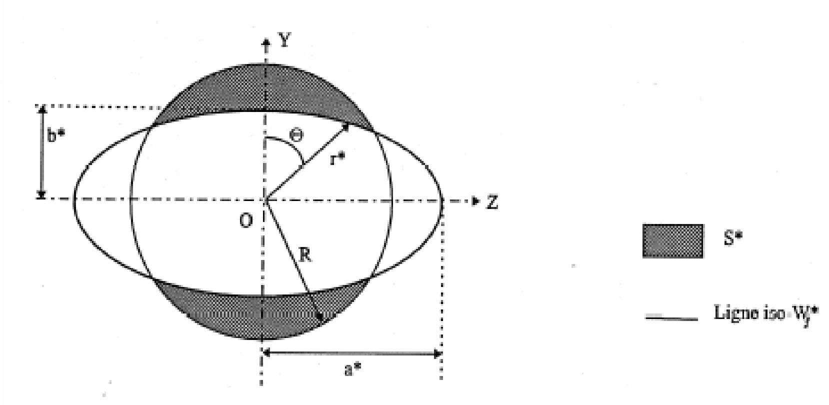


Fig 1.2 Iso-energetic level on the cross-section of a cylindrical smooth test-tube requested in rotational bending

The application of the postulate of volume of influence (1.3) expressing the equality $\varpi^D_{Trac} = \varpi^D_{FRot}$ and with the following limiting conditions (1.8) lead us to the energy variation of deformation provided expressed according to the threshold stress:

$$A_{Rot}\left(\sigma^{*\,2}_{Eq,Dissy}\right)^2 - B_{Rot}\left(\sigma^{*\,2}_{Eq,Dissy}\right)+C_{Rot} = 0 \quad (1.9)$$

A_{Rot}, B_{Rot} and C_{Rot} are evaluated by the limit of experimental endurance in traction, rotational bending and cross-bending

3.2 Cross-bending and torsion combined.

We consider that $\sigma^2_{eq,Fpla} = \sigma^2_{m,Fpla} + \sigma^2_{Fpla}$, and $\tau^2_{eq,Tor} = \tau^2_{m,Tor} + \tau^2_{Tor}$

The postulate of the volume of influence expressing the equality: $\varpi^D_{Trac} = \varpi^D_{Fpla}$.

In extreme cases of endurance is written: $\varpi_{fTrac} = \frac{1}{E}\left(\sigma^2_{Trac} - \sigma^{*\,2}_{Dissy}\right)$.

In the case of the loading checking the condition $\frac{a^*}{R} < 1$, Banvillet and al [4] show that:

$$\varpi_{fFrot} = \frac{1}{4E\left[1-\left(\frac{b^*}{R}\right)\left(\frac{a^*}{R}\right)\right]}\left\{\begin{array}{l}(2(1+\nu)\tau^2_{eq,Tor})\left(\frac{b^*}{R}\right)\left(\frac{a^*}{R}\right)^3 + (\sigma^2_{eq,Fpla} + 2(1+\nu)\tau^2_{eq,Tor})\left(\frac{b^*}{R}\right)^3\left(\frac{a^*}{R}\right) \\ +(\sigma^2_{eq,Fpla} + 4(1+\nu)\tau^2_{eq,Tor})\end{array}\right\}$$
$$-\frac{2\sigma^2_{Trac,-1} - \sigma^2_{Frot,-1}}{E}$$

(1.10)

The postulate of the volume of influence considered for a material: $W^*_{Trac} = W^*_{Fpla}$ allows to write:

$$\frac{\sigma^2_{m,Trac} + \sigma^{*\,2}_{Dissy}}{(\sigma^2_{m,Fpla} + \sigma^2_{Fpla})\cos^2\theta} = \left(\frac{r^*}{R}\right)^2 \qquad (1.11)$$

With the image of the case of loading of rotational bending dissymmetrical, the Iso-energetic level is described by an ellipse on the cross-section of the test-tube (Fig. 1.3). The small half axis a^* and the half large axis b^* of this ellipse are defined by the equations (1.11).

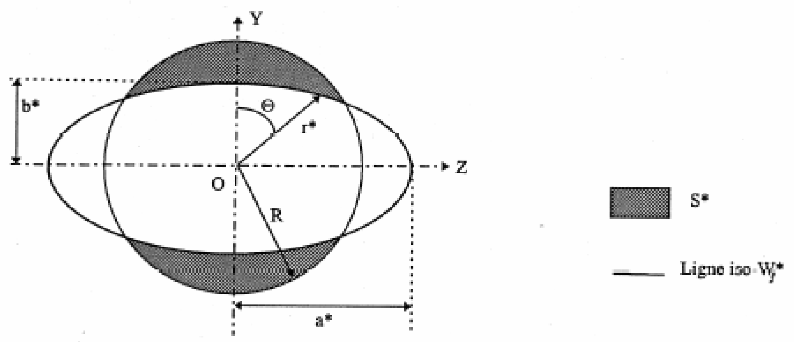


Fig 1.3 Iso-energetic level on the cross-section of a cylindrical smooth test-tube requested in cross-bending

The application of the postulate of volume of influence (1.3) expressing the equality $\varpi^D_{Trac} = \varpi^D_{Fpla}$ and with the following limiting conditions lead (1.12) us to the energy variation of deformation provided expressed according to the threshold stress.

$$A_{Fpla}\left(\sigma^{*\,2}_{Eq,Dissy}\right)^2 - B_{Fpla}\left(\sigma^{*\,2}_{Eq,Dissy}\right) + C_{Fpla} = 0 \qquad (1.12)$$

A_{Fpla}, B_{Fpla} and C_{Fpla} are evaluated by the limit of experimental endurance in traction, rotational bending and cross-bending

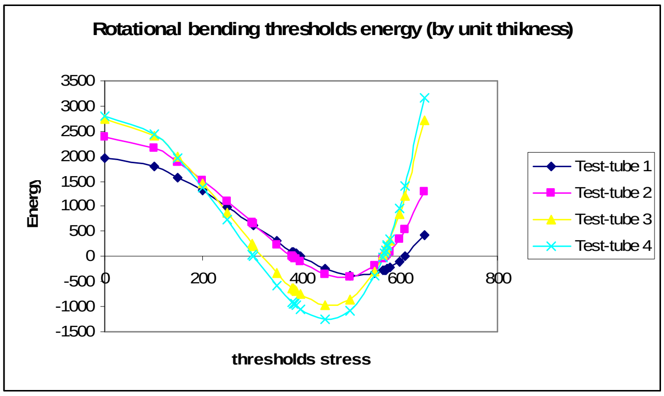
4 THE ENERGY VARIATION ANALYSE OF THE ROTATIONAL BENDING AND CROSS BENDING

In this paragraph we exploit the results of the fatigue tests to 10^6 of inflection carried out on the steel 30NCD16 obtained by C Froustey and Dubar [3]. The limit of experimental endurance in traction of the treated steel 30NCD16 comes from tests carried out by Galtier [7] is 560 MPa. To identify the thresholds stresses, the values 527 MPa, 222 MPa of the dynamic amplitude and the average of axial stress, in extreme cases of endurance are test results of Banvillet [4].

Work of De Roo and Clément [3] under uniaxial loading showed the appearance of privileged zones of starting of microscopic cracks in cast iron GS. In addition, the microscopic observations Pallin-Luc [8] on the appearance of the microscopic cracks and their evolution, under request of Cross-bending symmetrical, are definitely different under a loading 180 MPa from those under the loadings of amplitudes 230,268 or 280 MPa damage of cast iron GS61 as of the first 50000 cycles. These observations do not distinguish initiating and propagation from a threshold of microscopic cracks. This defect of method is consequently regulated by the global energy solution.

4.1 Rotational Bending and torsion combined

These tests are carried out on test-tubes for which the limit in symmetrical alternate torsion is 405 MPa and the limit in rotational bending of 585 MPa. Using the various configurations of rotational bending and torsion combined with average stress of rotational bending and/or torsion null or not, we can represent the evolution of the energy variation.

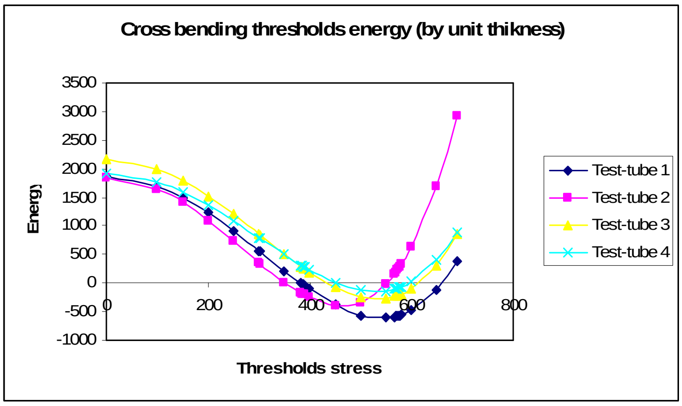


The branches of the curve constitute the energy provided to material, while the concave part limited by the two roots of the equation of the threshold stress interprets the energy released by the test-tube. Thus, the analysis of the equation (1.9) enables us to evaluate successively two stresses thresholds responsible for the initiating crack and the propagation of the maximum density of the microscopic cracks leading the test-tubes to the rupture.

	Test-tube 1	Test-tube 2	Test-tube 3	Test-tube 4
Initiating stress	330,9853462	310,5285011	231,178521	205,2630063
Propagation stress	569,8031168	527,3139957	520,199152	521,8765163

4.2 Cross-bending and torsion combined

The whole of the tests is carried out on test-tubes for which the limits of endurance in torsion and cross-bending alternate symmetrical are respectively 410 MPa and 660 MPa. Using the various configurations of cross-bending and torsion combined with average stress of cross-bending and/or torsion null or not, we can represent the evolution of the energy variation.



The branches of the curve constitute the energy provided to material while the concave part limited by two points of the axis of the threshold stress interprets the energy released by the test-tube. Thus, the analysis of the equation (1.12) enables us to locate successively two thresholds stresses responsible for the initiating crack and the propagation of the maximum density of the microscopic cracks leading material to the rupture.

	Test-tube 1	**Test-tube** 2	**Test-tube** 3	**Test-tube** 4
Initiating stress	310,9952551	272,9572975	371,34184	391,8565194
Propagation stress	621,9282897	505,554638	573,701894	551,7680544

5 CONCLUSIONS

In this paper the analytical energy approach study, highlights by using experimental results of the rotational bending or cross-bending combined with torsion, two threshold stresses.

Indeed, the biquadratic equation expressed (1.9) or (1.12) according to the threshold stress led to the assessment of initiating crack and crossing of the maximum density of the microscopic cracks. In the same way, we note through the analytical results that the extent, between the lower threshold stress of the reliable zone and the higher threshold stress after which the material reaches the endurance stress, varies between two hundred and three hundred MPa. This work has the advantage of distinguishing the value at the initiating crack from the value at the propagation crack.

Although the tests relate to same material, we note dispersion on the threshold stresses. Moreover, we note that the average stress delays the initiating of the microscopic cracks.

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